

Power spectra of scalar and tensor modes in modified Hořava-Lifshitz gravity

Bin Chen^{*}, Shi Pi[†] and Jin-Zhang Tang^{‡1}

¹*Department of Physics, and State Key Laboratory of Nuclear Physics and Technology,
Peking University, Beijing 100871, China*

(Dated: October 2, 2009)

Abstract

In Hořava-Lifshitz gravity, the extra dynamical scalar mode could play significant role in cosmology. However it has been pointed out that such a scalar may suffer from the strong coupling problem in IR. We address this issue in this paper. Our analysis shows that the scalar mode could decouple naturally at $\lambda = 1$ due to the extra gauge symmetry. On the other hand, the fact that the scalar mode becomes ghost when $1/3 < \lambda < 1$ is a real challenge to the theory. We try to overcome this problem by modifying the action such that the RG flow lies outside the problematic region. We discuss the cosmological implications of the action and calculate the power spectra of scalar and tensor modes.

PACS numbers: 98.80.Cq

^{*} Electronic address: bchen01@pku.edu.cn

[†] Electronic address: spi@pku.edu.cn

[‡] Electronic address: JinzhangTang@pku.edu.cn

I. INTRODUCTION

Diffeomorphism is essential to Einstein's relativity theory of gravity. It has been widely believed to be exact in any theory of gravity. However, in the recent proposal by Hořava[1, 2] on gravity theory, it is no longer an exact symmetry. The basic idea behind Hořava's theory is that time and space may have different dynamical scaling in UV limit. This was inspired by the development in quantum critical phenomena in condensed matter physics, with the typical model being Lifshitz scalar field theory[3, 4]. In this Hořava-Lifshitz theory, the time and space will take different scaling behavior as

$$\mathbf{x} \rightarrow b\mathbf{x}, \quad t \rightarrow b^z t, \quad (1)$$

where z is the dynamical critical exponent characterizing the anisotropy between space and time. Due to the anisotropy, instead of diffeomorphism, we have the so-called foliation-preserving diffeomorphism. The transformation is now just

$$\begin{aligned} t &\rightarrow \tilde{t}(t) \\ x^i &\rightarrow \tilde{x}^i(x^j, t). \end{aligned} \quad (2)$$

As the result of this “reduced” gauge symmetry, there are more physical degrees of freedom in the theory. In fact, there exist an extra dynamical scalar degree of freedom in Hořava-Lifshitz gravity, as shown in [1, 2]. This scalar degree of freedom and its physical implication has also been discussed in cosmology [5, 6, 31]. However, it was pointed out in [7] that this scalar mode would be strongly coupled to the matter in the IR fixed point $\lambda = 1$, at which one would expect the recovery of diffeomorphism. If this is the case, it would lead to unacceptable effects in many experiments. In the next section, we would like to show that this would not happen.

The key point in our analysis is that we take the point of view that the diffeomorphism is only an approximate symmetry even at IR. In fact, as we will review shortly, it is not hard to see that even at IR fixed point, there exist various other terms, involving spatial higher-derivative terms, which break the diffeomorphism, even though they should be very much suppressed. This is very different from the case in Fierz-Pauli's massive gravity[8]. In the massive gravity theory, there exist extra physical degree of freedom. It was a serious issue on how this degree of freedom get decoupled in the massless limit, where the diffeomorphism is completely recovered[9, 10]. In our case, we will show manifestly that the extra scalar degree of freedom could be decoupled without trouble, due to the existence of extra gauge symmetry at IR fixed point rather than the complete recovery of diffeomorphism. The breakdown of full diffeomorphism at IR fixed point also suggest that the usual Stuckelberg trick could not be used directly, especially taking into account of the projectability condition.

Another issue on the scalar mode is whether it is a real physical degree of freedom. The debate in the literature focus on if one should choose the lapse function to be only the function of time, or in other words, if the lapse function should be projectable. The different choice seems lead to completely different physics. For example, it was found that without the projectability condition there were new static spherically symmetric solutions to Hořava-Lifshitz gravity and its modifications[11, 12]. These new solutions may have profound physical implications in solar system tests[14]. However, it was proved in [13] that these new solutions do not respect the projectability condition. From our point of view, taking the lapse function as the function of time is the most natural choice. With this

choice, the gauge transformation looks transparent and simpler. Moreover the Hamiltonian constraints form a closed algebra, and the theory gets rid of the pathology found in [1, 15]. Simply speaking, the theory is well-defined with the projectability condition. As a result, the extra scalar mode becomes the physical one and the key ingredient in our following discussion.

On the other hand, this extra scalar mode could not be always physical. When the parameter λ lies between $1/3$ and 1 , this mode is actually a ghost. We would like to emphasize that the existence of the ghost is the real challenge to the original Hořava-Lifshitz gravity theory. It indicates that the theory may neither well-defined at UV, nor UV complete. In particular, at UV, one wish that the theory becomes non-relativistic and the speed of light is much larger than the constant one at IR. In the original proposal of Hořava-Lifshitz gravity, this requires that the theory stay near $\lambda = 1/3$. However in IR, one may expect that the theory would flow to $\lambda = 1$. As the RG flow is from $\lambda = 1/3$ to $\lambda = 1$, the theory inevitably suffers from the existence of ghost. To get away from this trouble, we try to modify the action in a way that the RG flow may be from UV with $\lambda > 1$ to IR with $\lambda = 1$. To simplify the analysis, instead of considering the most general form of the action, we only consider the potential with the marginal terms and the most relevant terms. This will be the topic in section 3.

In the remaining part of this paper, we discuss the cosmological implications of the scalar mode, and also calculate the power spectra of scalar and tensor perturbations. The cosmology of the Hořava-Lifshitz gravity has first been discussed in [11, 17, 18, 19], and then widely studied in the literature[20] from various angles. In this paper, we take the extra scalar as an alternative to the inflaton and study its power spectrum. We also calculate the power spectrum of tensor mode in modified Hořava-Lifshitz gravity action proposed in section 3. In our treatment, we simply ignore the RG flow and use the standard technology in cosmological perturbation theory. The problem turns out to be quite similar to the trans-Planckian problem in inflation. Instead of the WKB approximation used in [6, 19, 30], we apply the technology in trans-Planckian physics and study the equation of motion of scalar perturbation stage by stage. We find that the power spectra are scalar invariant. This is not a surprise since the classical evolution is a pure de-Sitter phase, which has time translation invariance. We also notice that the tensor-to-scalar ratio is sensitive to the time of horizon-crossing of tensor and scalar modes, and can be small if at the time of scalar crossing the horizon λ is near 1.

The paper is organized as follows. In section 2, we study the gravitational scalar in the Hořava-Lifshitz gravity theory. In section 3, we present our modification of the Hořava-Lifshitz gravity action. In section 4 and 5, we calculate the power spectra of the scalar and tensor perturbations respectively. We end with some discussions in section 6.

II. SCALAR MODE IN HOŘAVA-LIFSHITZ GRAVITY

Since time direction plays a privileged role in the whole construction, it is more convenient to work with ADM metric

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt). \quad (3)$$

Due to the anisotropy between time and space, the usual diffeomorphisms reduce to the foliation-preserving diffeomorphisms, generated by infinitesimal transformation:

$$\delta t = \xi^0(t), \quad \delta x^i = \xi^i(t, \vec{x}). \quad (4)$$

The essential point is that ξ^0 is just the function of t . This leads to the following transformations on the metric components:

$$\begin{aligned}\delta g_{ij} &= \partial_i \xi^k g_{jk} + \partial_j \xi^k g_{ik} + \xi^k \partial_k g_{ij} + \xi^0 \dot{g}_{ij}, \\ \delta N_i &= \partial_i \xi^j N_j + \xi^j \partial_j N_i + \dot{\xi}^j g_{ij} + \dot{\xi}^0 N_i + \xi^0 \dot{N}_i, \\ \delta N &= \xi^j \partial_j N + \dot{\xi}^0 N + \xi^0 \dot{N}.\end{aligned}\tag{5}$$

The above transformations could be obtained by taking a nonrelativistic limit of usual relativistic diffeomorphisms. It is more convenient and natural to choose N being just the function of t . There are a few advantages to work with this choice. With this choice, the gauge symmetry is simpler and transparent. Furthermore, in the Hamiltonian formulation, the constraints could form a closed algebra since the momentum conjugate to N does not lead to a local constraint[1]. As a result of less constraints than standard GR, the physical degrees of freedom in the theory include not only the massless gravitons but also another propagating scalar. The existence of extra scalar field has profound meaning in cosmology. In [6], we showed that for the action without the detailed balance condition, this scalar may lead to scale invariant spectrum.

On the other hand, if one abandon the projectability condition and let N be the function of both t and x^i , one will find that the theory would be ill-defined, as shown in [1, 15].

At the special value $\lambda = 1$, the theory develops an enhanced time-independent $U(1)$ gauge symmetry acting via

$$\delta N_i = \partial_i \epsilon, \quad \delta g_{ij} = 0.\tag{6}$$

Due to the existence of extra gauge symmetry, the scalar mode is not physical anymore. It is remarkable that even with this extra gauge symmetry, the total gauge symmetries is different from the usual diffeomorphisms in general relativity. In other words, the diffeomorphisms has not been recovered at $\lambda = 1$. This fact is essential to understand why at $\lambda = 1$ the extra scalar degree of freedom could be decoupled without trouble.

To understand the decoupling better, let us consider the perturbation around FRW metric:

$$\begin{aligned}ds^2 &= -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j \\ &= a^2(\eta)(-d\eta^2 + \delta_{ij} dx^i dx^j).\end{aligned}\tag{7}$$

Here for simplicity we focus on the flat universe, and use the co-moving time $\eta = \int dt/a$ as time variable. The above metric could be reduced to Minkovski spacetime if $a(t)$ is a constant. Perturb the flat metric, and use the ADM formulism in co-moving time,

$$ds^2 = -(\mathcal{N}^2 - \mathcal{N}^i \mathcal{N}_i) d\eta^2 + 2\mathcal{N}_i d\eta dx^i + g_{ij} dx^i dx^j,\tag{8}$$

The fluctuations around the above metric could be

$$\mathcal{N} = a(\eta)(1 + A),\tag{9}$$

$$\mathcal{N}_i = a(\eta)(\partial_i B + V_i),\tag{10}$$

$$g_{ij} = a^2(\eta)\{(1 - 2\psi)\delta_{ij} - \partial_i \partial_j E - 2\partial_{(i} F_{j)} + h_{ij}\},\tag{11}$$

where A, B, ψ, E are scalar perturbations, V_i and F_j are vector perturbations, and t_{ij} is the gauge-invariant tensor perturbation describing gravitational wave. Under the gauge

transformations (5), we have

$$A \rightarrow \tilde{A} = A - \frac{1}{a}(\xi^0 a)', \quad (12)$$

$$B \rightarrow \tilde{B} = B - a\zeta', \quad (13)$$

$$E \rightarrow \tilde{E} = E + 2\zeta, \quad (14)$$

$$\psi \rightarrow \tilde{\psi} = \psi + \xi^0 \frac{a'}{a}, \quad (15)$$

$$V_i \rightarrow \tilde{V}_i = V_i + (\xi_{i\perp})', \quad (16)$$

$$F_j \rightarrow \tilde{F}_j = F_j + (\xi_{i\perp})', \quad (17)$$

$$h_{ij} \rightarrow \tilde{h}_{ij} = h_{ij}, \quad (18)$$

where we have decompose the spatial vector ξ^i as

$$\xi^i = \xi_{\perp}^i + \partial^i \zeta \quad (19)$$

with ξ_{\perp}^i being divergenceless and ζ being a scalar. The gauge invariant variables besides h_{ij} are

$$\Psi = A + \psi + \left(\frac{\psi}{\mathcal{H}} \right)', \quad (20)$$

$$\Phi = B + \frac{E'}{2}, \quad (21)$$

$$S_i = V_i - F_i. \quad (22)$$

We will only focus on the scalar perturbations. It is convenient to work with the gauge

$$A = 0, \quad E = 0. \quad (23)$$

Note that the above gauge choice is consistent with the projectability condition. Since ξ^0 is the only function of t , the gauge transformation on A would not spoil the projectability condition.

If the scale factor is a constant, the above gauge transformations reduce to the ones in flat spacetime. In [16], the gauge invariant perturbations about the flat spacetime have been analyzed carefully. Actually, from the discussion there, one can see that the extra dynamical scalar mode can decouple without trouble. It was claimed in [16] that such scalar mode is not a propagating mode. This is not true. The problem comes from the fact that the lapse function is projectable so that it induce a non-local super-Hamiltonian constraint. For the flat spacetime, the perturbation A of the lapse function is a pure gauge and can be set to zero safely. Even if A is kept nonvanishing, the variation with respect to A would only lead to non-local constraint, which is less powerful than the local one.

In terms of ADM metric, the action of original Hořava-Lifshitz gravity theory can be written as[2]

$$S_g = \int dt d^3 \mathbf{x} \sqrt{g} N \left\{ \frac{2}{\kappa^2} K_{ij} G^{ijkl} K_{kl} - \frac{\kappa^2}{2} \left[\frac{1}{\omega^2} C_{ij} - \frac{\mu}{2} \left(R_{ij} - \frac{1}{2} R g_{ij} + \Lambda_W g_{ij} \right) \right] \right. \\ \left. \cdot G^{ijkl} \left[\frac{1}{\omega^2} C_{ij} - \frac{\mu}{2} \left(R_{ij} - \frac{1}{2} R g_{ij} + \Lambda_W g_{ij} \right) \right] \right\}. \quad (24)$$

where K_{ij} is the extrinsic curvature of the spatial hypersurface; C_{ij} is the Cotton tensor which can be used to preserve the detailed-balanced condition in constructing the action; G^{ijkl} is the De Witt metric on the space of metrics that preserve the anisotropic diffeomorphism, and R_{ij} is the Ricci tensor in spatial hypersurface. Their definitions are

$$K_{ij} = \frac{1}{2N}(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i), \quad (25)$$

$$C_{ij} = \epsilon^{ikl} \nabla_k \left(R^j_l - \frac{1}{4} R \delta^j_l \right), \quad (26)$$

$$G^{ijkl} = \frac{1}{2} (g^{ik} g^{jl} + g^{il} g^{jk}) - \lambda g^{ij} g^{kl}. \quad (27)$$

Here and throughout the paper, a dot over the quantity means taking the derivative with respect to cosmic time t , while a prime denotes that to co-moving time η . The first term in (24) involving only the extrinsic curvature is the kinetic term, while the others are potential terms. λ is the coupling constant in the kinetic term, and runs expectedly to $\lambda = 1$ at IR regime at which the kinetic term goes back to the one in the general relativity. This specific form of the action is governed by the detailed-balance condition, which is just applied by Hořava for convenience to decrease the number of arbitrary parameters. The expansion of the action gives

$$S_g = \int dt d^3 \mathbf{x} \sqrt{g} N \left\{ \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) - \frac{\kappa^2}{2\omega^4} C_{ij} C^{ij} + \frac{\kappa^2 \mu}{2\omega^2} \epsilon^{ijk} R_{il} \nabla_j R^l_k - \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij} + \frac{\kappa^2 \mu^2}{8(1-3\lambda)} \left(\frac{1-4\lambda}{4} R^2 + \Lambda_W R - 3\Lambda_W^2 \right) \right\}. \quad (28)$$

Comparing this action with the Einstein-Hilbert action in IR limit

$$S_{\text{EH}} = \frac{1}{16\pi G} \int d^4 x \sqrt{g} N \{ (K_{ij} K^{ij} - K^2) + R - 2\Lambda \}, \quad (29)$$

with $x^0 \equiv ct$, we can recover the speed of light, Newton constant and the cosmological constant by the parameters introduced before,

$$c = \frac{\kappa^2 \mu}{4} \sqrt{\frac{\Lambda_W}{1-3\lambda}}, \quad G = \frac{\kappa^2}{32\pi c}, \quad \Lambda = \frac{3}{2} \Lambda_W. \quad (30)$$

Thus at IR the theory recovers nearly the usual general relativity, with the higher derivative terms of spatial metric components as the modifications. Even though the higher derivative terms are highly suppressed at IR, strictly speaking, the theory always breaks diffeomorphism, or locally Lorentz invariance.

In Hořava's original paper [2] the coupling constant λ runs to 1 in IR limit. And in UV, because of the anisotropy between space and time, the speed of light is not a constant and may be extremely large, which could be used to explain the horizon and flatness problem[18]. But from (30) we know that this can only occur in the case $\lambda < 1/3$ if we take Λ to be positive, taking into account of the fact Λ is directly related to cosmological constant. However, this raise the worry that the marginal coupling constant λ can never run to its infrared value $\lambda = 1$, which is directly in contrast with our former description. To solve this problem, it was proposed that one should do analytical continuation on the parameters[11]

$$\mu \rightarrow i\mu, \quad \omega \rightarrow -i\omega, \quad (31)$$

which leaves the action real. And under this continuation, we see from (30) that

$$c = \frac{\kappa^2 \mu}{4} \sqrt{\frac{\Lambda_W}{3\lambda - 1}}, \quad (32)$$

and there is no conflict between $\Lambda > 0$ and $\lambda > 1/3$. And when $\lambda \rightarrow 1/3$ proposed by Hořava as the ultraviolet value of this coupling constant, we have a very large speed of light, which can naturally solve the causality problem in cosmology without inflation.

To understand the extra scalar degree of freedom, let us come back to the scalar perturbation we studied before [5, 6]. We need only focus on the kinetic term in the above action, which is just

$$S_K = \int dt d^3 \mathbf{x} \left\{ 3\alpha a^3 (1 - 3\lambda) \left[\frac{2}{3} \frac{\dot{\psi}^2}{1 - \lambda} + 6H\psi\dot{\psi} + 9H^2\psi^2 \right] \right\}. \quad (33)$$

Several remarks are in order:

1. From the action, it is obvious that the scalar mode ψ is physical when $\lambda < 1/3$ and $\lambda > 1$, while when $1/3 < \lambda < 1$ the mode is a ghost, indicating that the theory is not well-defined [21]. At the special value $\lambda = 1$, the mode is decoupled, as we will clarify more below. And at $\lambda = 1/3$, the theory has extra symmetry, as discussed carefully in [2]. This fact is the same as the one found in [1, 2] where the perturbations around the flat spacetime were studied.

2. More interestingly, the equation of motion of ψ takes the following form:

$$\frac{1 - 3\lambda}{1 - \lambda} \ddot{\psi} + \dots \quad (34)$$

This indicates that when $\lambda \rightarrow 1$, the scalar field ψ could be decoupled naturally, in contract with the claim in [7]. It seems that the strong coupling problem does not exist in our case.

3. The absence of the strong coupling problem may stem from the fact that we take different points of view on gauge transformations. In our case, we stick to the requirement that the lapse function should be projectable, as originally advocated in [2]. As a result, we do not expect that the diffeomorphism is recovered at $\lambda = 1$. Instead, the decoupling of the extra scalar mode comes from the fact that there is extra gauge symmetry at $\lambda = 1$. This is conceptually different from the case studied in [7] and Fierz-Pauli massive gravity [8].
4. Technically it is remarkable the equation of motion of ψ has a prefactor proportional to $1/(1 - \lambda)$ rather than $(1 - \lambda)$. This difference has significant physical implication. In our case, this means that the scalar mode could be decoupled without trouble. Another way to see this is to cast the scalar mode into canonical form such that the mode become non-physical at $\lambda = 1$. It is remarkable that in [1, 2], the equation of motion of the scalar mode around the flat spacetime background has the prefactor $(1 - \lambda)$. However this is due to different gauge choice. It has been shown in [16] by rescaling the field, one has the same equation of motion. In fact, no matter what kind of gauge choice, the physical dispersion relation is exactly the same. This suggests that for the cosmological perturbations, the different gauge choice would not lead to different dispersion relation. Namely, the extra scalar mode may decouple naturally as $\lambda \rightarrow 1$.

III. MODIFIED HOŘAVA-LIFSHITZ GRAVITY

The existence of the ghost is fatal to the theory. It means that the theory is not well-defined, not mentioning UV completeness. One may expect that we can always work in the region outside $\lambda \in [1/3, 1]$. However this cannot be guaranteed, considering our ignorance of the details of RG flow. On the other hand, in the practical application in cosmology, one wish the RG flow is from $\lambda \sim 1/3$ to $\lambda = 1$ in original Hořava-Lifshitz gravity. In this paper, we take a modest attitude and try to modify the Hořava-Lifshitz gravity such that the RG flow may happen always with $\lambda > 1$. In order to do so, we have to abandon the detailed balance condition. As it is well-known, the detailed balance condition may not be essential to the theory[26, 28, 33]. The imposing of such condition is pragmatic to simplify the action. In principle, one may relax this condition and consider more general form of the action. In this paper, we do not want to consider the most general form of the action. Instead, we just consider the marginal spatial kinetic part and most relevant deformations, besides the time kinetic terms. The action we start with is of the form

$$S_g = \int dt d^3\mathbf{x} \sqrt{g} N \left\{ \alpha (K_{ij} K^{ij} - \lambda K^2) + \xi(\lambda) R + \sigma(\lambda) \right. \\ \left. - \beta \left(\beta_1 \nabla_i R_{jk} \nabla^i R^{jk} + \beta_2 \nabla_i R_{jk} \nabla^j R^{ik} + \beta_3 \nabla_i R \nabla^i R \right) \right\}. \quad (35)$$

Here we only keep the marginal terms that are power-counting renormalizable and dominant in UV limit, besides the lower-dimensional terms to recover IR behaviors.¹ The other marginal terms being cubic of Ricci scalar and Ricci tensor, and the other relevant terms like R^2 and $R\nabla R$ are neglected for simplicity. For a complete discussion on all possible terms maintaining the power-counting renormalizability, see [26].

Because of the breakdown of the detailed balance condition, the coupling constants before each terms are independent. The couplings could be connected to the speed of light, the Newtonian coupling constant and the cosmological constant of Einstein's general relativity in IR limit,

$$c^2 = \frac{\xi}{\alpha}, \quad (36)$$

$$16\pi G = \frac{1}{c\alpha}, \quad (37)$$

$$\Lambda = -\frac{\sigma}{2\xi}. \quad (38)$$

Here we see that c^2 can be positive, if we choose a proper form of the function $\xi(\lambda)$. Furthermore, we can require c to be very large when λ is near its ultraviolet value. In Hořava's original paper, he suggested $\lambda \rightarrow 1/3$ at the UV limit, which gives a large speed of light in (30) or (32). Here we only take this condition as a constraint on the function $\xi(\lambda)$. For instance if the theory requires λ to be larger than the unity at UV as we will propose as a condition to exclude the ghost field, the function $\xi(\lambda)$ may be divergent when λ tends to be infinity.

¹ Actually, after some integrals by parts and using the Bianchi identity, the $\beta_2 \nabla_i R_{jk} \nabla^j R^{ik}$ term can be converted to a $(\beta_2/4) \nabla_i R \nabla^i R$ term and some higher order terms. In our current work we can just set $\tilde{\beta}_3 = \beta_3 + \beta_2/4$ and discard the β_2 term. This will change in the calculation of non-Gaussianity. Thanks Shinji Mukohyama for useful discussions.

For our use let us have a glance of the classical dynamics of the universe under such an action. In a homogenous and isotropic universe,

$$ds^2 = -dt^2 + a^2 h_{ij} dx^i dx^j, \quad h_{ij} = \delta_{ij} + \frac{\mathcal{K} x^i x^j}{1 - \mathcal{K} \mathbf{x}^2}, \quad (39)$$

where \mathcal{K} is the parameter to describe the spatial curvature. Under this metric, the universe is homogeneous and isotropic, which will greatly simplify our following calculations. To apply our foliated diffeomorphism, we need use the ADM formalism of this Robertson-Walker metric, with the extrinsic curvature and the Ricci tensor to be

$$K_{ij} = H(t) g_{ij}, \quad K = 3H(t), \quad (40)$$

$$R_{ij} = \frac{2\mathcal{K}}{a^2} g_{ij}, \quad R = \frac{6\mathcal{K}}{a^2}, \quad (41)$$

where $H(t) = \dot{a}/a$ is the Hubble parameter.

We take the variation of the action (35) with respect to N , and have our first equation of constraint.

$$\int \sqrt{g} \left[-\frac{2}{\kappa^2} (K^{ij} K_{ij} - \lambda K^2) + \zeta R + \sigma \right] d^3 \mathbf{x} = \int \sqrt{g} \rho d^3 \mathbf{x}. \quad (42)$$

Here, ρ is the energy density of the Lifshitz scalar in the universe, and can be written as

$$\rho = -\frac{1}{\sqrt{g}} \frac{\delta S_m}{\delta N} \quad (43)$$

where S_m is the action of matter field, which can be a Lifshitz scalar[17], gauge field[4] or something else. Because of the projectability of the lapse function $N(t)$, we only have a spatial-integral constraint here. This is generic for all the Hořava-like models with projectable lapse function $N(t)$ [26]. But, for a homogeneous and isotropic Friedman universe, this constraint equation is valid at every point, and the integral can be removed legally. Thus we have the first Friedman's equation[11, 17]

$$H^2 = \frac{c^2}{3\alpha(1-3\lambda)} \left[-\rho + \frac{6\mathcal{K}}{a^2} \xi(\lambda) + \sigma(\lambda) \right]. \quad (44)$$

Since ρ is the energy density of matter and radiation, $\sigma(\lambda)$ plays the role of “cosmological constant”. Here, it is a function of λ and evolves when λ varies as the energy scale changes. This implicant dependence may be treated carefully when we are facing problems like the evolution of dark energy or the tilt of the power spectrum. But because the dependence of λ on the cosmic time is unknown, we will neglect this dependence and suppose that in the process we are interested in, the change of $\sigma(\lambda)$ is so little that it will not have any significant physical effect, and so is $H(\lambda)$. We see from (44) that if the universe is flat and dominated by the cosmological constant, for some λ greater than $1/3$, we must have $\sigma(\lambda > 1/3) < 0$, which means that we have a positive cosmological constant $\Lambda > 0$ at IR, since from (36), $\xi(\lambda > 1/3)$ is always positive. These two conditions guaranteed the positivity of the cosmological constant and H^2 . If the matter/radiation contribution could be ignored safely, the homogenous and isotropic solution is a pure de-Sitter spacetime, with an exponentially expanding scale factor $a(t) \propto \exp(Ht)$.

The second equation of constraint is obtained by taking the variation of the action with respect to the shift vector N_i ,

$$\nabla_i(K^{ij} - \lambda K g^{ij}) = 0. \quad (45)$$

Because the extrinsic curvature is homogeneous in a Friedman universe, as in (40), $K_{ij} \propto g_{ij}$, this equation is trivially satisfied for the background evolution. But it will supply a perturbative constraint equation up to first order if the perturbations to the background metric are under consideration.

Finally take the variation of action (35) with respect to g_{ij} , we have the equation of motion of dynamical degree of freedom. The explicit expression (see [11, 18, 26]) is rather lengthy and has little to do with our following discussion so we would like not write it here.

IV. SPECTRUM OF THE GRAVITATIONAL SCALAR

From the discussion above, we know that the classical evolution of the scale factor in the Hořava era is determined by (44). Especially, when cosmological constant is dominant and the universe is flat, the evolution is the exponentially expansion like in a de Sitter phase. In this section, we will calculate the perturbation of the gravitational field, and study the equations of motion of the scalar modes.

Taken the ADM formalism and the gauge choice (23), with some relations derived through the two constraint equations, the perturbed action of the gravitational field up to second order can be written as

$$S^{(2)} = \int dt d^3\mathbf{x} \left\{ 3\alpha a^3(1-3\lambda) \left[\frac{2}{3} \frac{\dot{\psi}^2}{1-\lambda} + 6H\psi\dot{\psi} + 9H^2\psi^2 \right] - \frac{2\beta}{a^3} (3\beta_1 + 2\beta_2 + 8\beta_3) \psi \partial^6 \psi - 2a\xi(\lambda) \psi \partial^2 \psi \right\}. \quad (46)$$

Now the Hubble parameter is a constant. For convenience we define a conformal time η with $dt = a d\eta$ and introduce an auxiliary field $\chi = a\psi$. After taking the variation with respect to ψ , and changing to the momentum space, we have

$$\chi''(\eta) + \left(k^6 H^4 \bar{L}^4 \eta^4 + c_s^2 k^2 - \frac{2}{\eta^2} \right) \chi(\eta) = 0. \quad (47)$$

where

$$c_s^2 = \frac{1-\lambda}{1-3\lambda} c^2 \quad (48)$$

is the speed of sound, and

$$\bar{L} = \frac{L}{2\pi}, \quad L = 2\pi \left[\frac{\beta}{\alpha} \frac{1-\lambda}{1-3\lambda} (3\beta_1 + 2\beta_2 + 8\beta_3) \right]^{\frac{1}{4}}, \quad (49)$$

is the characteristic length which denotes the scale where the trans-Planckian effects becomes significant.

This equation can not be solved analytically. However, many efforts has been done to deal with this type of Corley-Jacobson dispersion relation[22, 23, 24]. The author of [29] have already studied the trans-Planckian physics appearing naturally in Horava-Lifshitz

gravitational waves, and find a scale invariant power spectrum in a specific connecting time. Here we follow the method of Martin et.al.[22] to investigate the trans-Planckian effects of the gravitational scalar. We split the period under consideration into three regions by two different characteristic lengths: the trans-Planckian length L and the sound horizon c_s/H . First, when $\eta \rightarrow -\infty$, we can set the initial value of the wave function $\chi(\eta_i)$ and its derivative $\chi'(\eta_i)$ such that they initially minimize the energy density. This is to satisfy

$$\chi(\eta_i) = \frac{1}{\sqrt{2k^3}} \frac{1}{H\bar{L}|\eta_i|}, \quad (50)$$

$$\chi'(\eta_i) = \pm i \sqrt{\frac{k^3}{2}} H\bar{L}|\eta_i|. \quad (51)$$

Then, in UV region when the physical wavelength of the mode concerned is much smaller than the characteristic length, k^6 term is dominant in (47),

$$\chi''_{\text{UV}} + k^6 H^4 \bar{L}^4 \eta^4 \chi_{\text{UV}} = 0. \quad (52)$$

In a new variable $z = k^3 H^2 \bar{L}^2 |\eta|^3 / 3$, the solution to this equation can be expressed as

$$\chi_{\text{UV}}(\eta) = A_1 \sqrt{|\eta|} J_{1/6}(z) + A_2 \sqrt{|\eta|} J_{-1/6}(z), \quad (53)$$

where A_1 and A_2 can be determined by the continuity of χ and χ' at the initial time η_i , to be

$$A_1 = \frac{\pi H \bar{L}}{6 \sin(\pi/6)} \sqrt{\frac{k^3 |\eta_i|^3}{2}} [J_{5/6}(z_i) \mp i J_{-1/6}(z_i)], \quad (54)$$

$$A_2 = \frac{\pi H \bar{L}}{6 \sin(\pi/6)} \sqrt{\frac{k^3 |\eta_i|^3}{2}} [J_{-5/6}(z_i) \mp i J_{1/6}(z_i)], \quad (55)$$

Note, that in the UV region $|\eta_i| \gg 1$ and $z_i \gg 1$, we can expand the Bessel's function into its asymptotic form when the argument is large, and the coefficients reads

$$A_1 \approx \sqrt{\frac{\pi}{3}} (\mp i) \exp \left[\mp i \left(z_i + \frac{\pi}{12} - \frac{\pi}{4} \right) \right] \equiv \pm i \sqrt{\frac{\pi}{3}} e^{\mp i x_i}, \quad (56)$$

$$A_2 \approx \sqrt{\frac{\pi}{3}} (\pm i) \exp \left[\pm i \left(z_i - \frac{\pi}{12} - \frac{\pi}{4} \right) \right] \equiv \pm i \sqrt{\frac{\pi}{3}} e^{\pm i y_i}. \quad (57)$$

Here for simplicity we have defined

$$x = z + \frac{\pi}{12} - \frac{\pi}{4}, \quad y = z - \frac{\pi}{12} - \frac{\pi}{4}. \quad (58)$$

In the intermediate region, the wavelength of the k -mode exceeds the characteristic length L but still much less than the sound horizon c_s/H , i. e. $L \gg \lambda \gg c_s H^{-1}$. Then we can neglect the UV term and the cosmological damping term in (47), and get an oscillation solution of the perturbation as a plane wave. We can deduce the connecting time η_* between

UV and intermediate region ²,

$$\frac{2\pi}{k}a(\eta_*) = L_*, \quad |\eta_*| = \frac{2\pi}{kHL_*} = \frac{1}{kH\bar{L}_*}. \quad (59)$$

Here, the emergence of the subscript \star of the characteristic length L_\star is because that L also varies with time through the parameter λ , which depends on the energy scale by the renormalization flow. When we calculate the critical time η_* , all the parameters should be the value at η_* including L . In this region, the solution is the usual plane wave

$$\chi_{\text{int}}(\eta) = B_1 e^{ic_s k \eta} + B_2 e^{-ic_s k \eta}. \quad (60)$$

Then we can determine the coefficients B_1 and B_2 by connecting χ_{int} and its derivative with those in the UV region. This reads

$$\begin{aligned} B_1 e^{ic_s k \eta_*} &= \frac{A_1}{2} |\eta_*|^{1/2} [J_{1/6}(z_*) + ik^2 H^2 \bar{L}^2 |\eta_*|^2 J_{-5/6}(z_*)] \\ &\quad + \frac{A_2}{2} |\eta_*|^{1/2} [J_{-1/6}(z_*) - ik^2 H^2 \bar{L}^2 |\eta_*|^2 J_{5/6}(z_*)], \end{aligned} \quad (61)$$

$$\begin{aligned} B_2 e^{-ic_s k \eta_*} &= \frac{A_1}{2} |\eta_*|^{1/2} [J_{1/6}(z_*) - ik^2 H^2 \bar{L}^2 |\eta_*|^2 J_{-5/6}(z_*)] \\ &\quad + \frac{A_2}{2} |\eta_*|^{1/2} [J_{-1/6}(z_*) + ik^2 H^2 \bar{L}^2 |\eta_*|^2 J_{5/6}(z_*)]. \end{aligned} \quad (62)$$

To go further we notice that

$$k^2 H^2 \bar{L}_*^2 |\eta_*|^2 = 1, \quad (63)$$

and generally $L_\star \ll H^{-1}$, so we also have $|\eta_i| \gg |\eta_*| \gg 1$. The Bessel functions can also be expanded in its asymptotic form as before, then we get

$$B_1 = \mp \frac{i}{\sqrt{2k}} e^{-ic_s k \eta_*} e^{\pm i x_i} [e^{-iy_*} - e^{\mp i\pi/6} e^{-ix_*}], \quad (64)$$

$$B_2 = \mp \frac{i}{\sqrt{2k}} e^{ic_s k \eta_*} e^{\pm i x_i} [e^{iy_*} - e^{\mp i\pi/6} e^{ix_*}], \quad (65)$$

where x and y are defined in (58).

As the universe expands the wavelength of k -mode is stretched and becomes larger and larger, and finally exceeds the sound horizon. This critical time η_* is determined by

$$\frac{a(\eta_*)}{k} = \frac{c_{s*}}{H}, \quad |\eta_*| = \frac{1}{c_{s*} k}. \quad (66)$$

The meaning of the subscript $*$ of the sound speed is similar as before: we require λ in the definition of c_s be its value at η_* . When $\eta \gg \eta_*$, the fluctuation is in an IR region. The perturbation will freeze out after it exceeds the horizon, so primordial value of the power

² Here some subtlety exists when choosing the exact connecting time whether we should let $\lambda = L$ or $k^6 H^4 \bar{L}^4 \eta^4 = c_s^2 k^2$. In [23], the detailed discussion shows that we should choose to avoid the oscillation spectrum. But in our case we will show that the former choice will also sweep the oscillation in the final result.

spectrum observed today can be traced back to its value at and before η_* . The IR solution of equation (47) is

$$\chi_{\text{IR}} = C\eta^2 + \frac{D}{\eta} \quad (67)$$

For convenience we only pick the increasing mode³, whose coefficient can be determined by connecting χ_{int} and χ_{IR} at η_* , which reads

$$D = \frac{1}{c_{s*}\sqrt{2k^3}} \exp \left\{ \mp i \left[k(\eta_* - \eta_*) + z_* - z_i - \frac{\pi}{2} \right] \right\}. \quad (68)$$

Therefore the power spectrum is

$$\mathcal{P}_\psi = \frac{k^3}{2\pi^2} |\psi|^2 = \frac{k^3}{2\pi^2} H^2 |D|^2 = \left(\frac{H}{2\pi c_s} \right)_*^2. \quad (69)$$

which is obviously scale-invariant, if we neglect the time variation of the horizon at the inflationary stage. If so, the slight difference of the horizon-crossing time for different wavelengths will produce different H 's, thus different spectra. This is of course the usual case in the model slightly breaking the time-translation invariance of the de-Sitter stage.

To comprehend the significance of (69), we note that the spectrum is frozen out after the mode exceeds the sound horizon. When the exponentially expansion is over and the universe recovers usual GR behavior, all the parameters in our original action (35) recover the IR limit value, and specifically, $\lambda \rightarrow 1^+$. But this will never influence the value of the power spectrum which is completely determined by the values of parameters at horizon-crossing. On the other hand, if λ runs to unity much earlier before the wavelength of the fluctuation mode exceeds the sound horizon, i. e. $c_s(\eta < \eta_*) = 0$,⁴ then the scalar spectrum is divergent, showing the breakdown of the treatment. This is just what we expect: the theory recovers the general relativity so early that it looks the same as the usual inflationary model and the gravitational scalar is not a physical degree of freedom and will not bring observable power spectrum any more.

V. PRIMORDIAL GRAVITATIONAL WAVES

From the discussion above we see that the scalar mode of the gravitational scalar is scale invariant. This is the generic property under the de Sitter background with time translation invariance. We will see the same result for the gravitational tensor modes[29]. We also start with the perturbed metric, only to the tensor parts,

$$ds^2 = -dt^2 + a^2(t) (\delta_{ij} + h_{ij}(t, \vec{x})) dx^i dx^j, \quad (70)$$

where h_{ij} has been already defined in 10, and satisfies the transverse-traceless conditions $h_i^i = 0$, $\partial_i h_{ij} = 0$. Substituting this metric into the total action, we obtain the tensor action of second order,

$$S_g^{(2)} = \int dt d^3x a^3 \left[\frac{\alpha}{4} \dot{h}_i^j \dot{h}_j^i + \beta_1 \frac{\beta}{4a^6} \Delta^2 h_i^j \Delta h_j^i + \frac{\xi(\lambda)}{4a^2} h_j^i \Delta h_i^j \right]. \quad (71)$$

³ Connect with both the decaying and increasing modes will bring here an inessential factor of order unity, as in [?].

⁴ This requires $c(\lambda)$ remains finite in the case when $\lambda \rightarrow 1$.

The transverse traceless tensor h_{ij} can be Fourier transformed by plane waves with wavenumber \mathbf{k} as

$$h_{ij}(t, \mathbf{k}) = \sum_{A=R,L} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \psi_{\mathbf{k}}^A(t) e^{i\mathbf{k}\cdot\mathbf{x}} p_{ij}^A, \quad (72)$$

where p_{ij}^A is the circular polarization tensor which is defined by $ik_s \epsilon^{rsj} p_{ij}^A = k \rho^A p_i^{rA}$ [25]. Here $\rho^R = 1$, $\rho^L = -1$, and are called the right handed mode and the left handed mode respectively. We also impose normalization conditions as $p_j^{*iA} p_i^{jB} = \delta^{AB}$, where p_j^{*iA} is the complex conjugate of p_j^{iA} . Substituting the expansion into the action (71), we obtain

$$\delta^2 S_g = \sum_{A=R,L} \int dt \frac{d^3\mathbf{k}}{(2\pi)^3} a^3 \left\{ \frac{\alpha}{4} |\dot{\psi}_{\mathbf{k}}^A|^2 - \left[\beta_1 \frac{\beta}{4a^6} k^6 + \frac{\zeta(\lambda)}{4a^2} \right] |\psi_{\mathbf{k}}^A|^2 \right\}. \quad (73)$$

Using the variable $v_{\mathbf{k}} \equiv a \psi_{\mathbf{k}}^A$ and the conformal time η , and rewrite the action by co-moving time η , with $a = -1/H\eta$ in the de Sitter-like space, we have

$$v_{\mathbf{k}}''(\eta) + \left(k^6 H^4 \bar{l}^4 \eta^4 + c^2 k^2 - \frac{2}{\eta^2} \right) v_{\mathbf{k}}(\eta) = 0. \quad (74)$$

where similarly to the treatment on the scalar perturbation before, we define a characteristic length related to ultraviolet gravitational waves, $\bar{l} = (\beta_1 \beta / \alpha)^{1/4}$. We see from this definition and (49), that the characteristic length of scalar and tensor modes may be different. This relies on the relative magnitude of different β 's.

Some discussions parallel to last section will yield the power spectrum of gravitational wave as

$$\mathcal{P}_h = \frac{k^3}{2\pi^2} \frac{|v_{\mathbf{k}}|^2}{a^2} = \left(\frac{H}{2\pi c} \right)_{\dagger}^2 \quad (75)$$

where the subscript \dagger means the quantities are evaluated at the time of horizon-crossing of the gravitational waves, i.e.

$$\frac{a(\eta_{\dagger})}{k} = \frac{c_{\dagger}}{H}, \quad |\eta_{\dagger}| = \frac{1}{c_{\dagger} k}. \quad (76)$$

Obviously the power spectrum of tensor mode is scale invariant as well. In [29], the author use the same method to connect the solution step by step and calculate the infrared power spectrum, but with only two pieces to join and thus more accurate sub-solution in each piece: Hankel function in infrared region. However, after taking the correct connecting time η_{\star} in (59) as we do before, the dependence on the ‘‘cutoff energy scale’’ there also vanishes.

Now we can calculate the tensor-to-scalar ratio⁵,

$$r = \frac{\mathcal{P}_h}{\mathcal{P}_{\psi}} = \frac{c_{s*}}{c_{\dagger}} = \frac{1 - \lambda_{*}}{1 - 3\lambda_{*}} \frac{\xi(\lambda_{*})}{\xi(\lambda_{\dagger})}. \quad (77)$$

⁵ Note that the tensor-to-scalar ratio is usually defined to be $\mathcal{P}_h / \mathcal{P}_{\delta\phi}$, by the power spectrum of the perturbations of inflaton field in ordinary inflationary models. And in super-horizon scales, $\mathcal{P}_{\delta\phi}$ is of the order as \mathcal{P}_h divided by slow-roll parameter ϵ . Because we have not placed any scalar field here, we define the tensor-to-scalar ratio as \mathcal{P}_h divided by just the spectrum of gravitational scalar

This shows that the ratio is only determined by the speed of sound/light at the sound-/hubble-horizon-crossing. All the dependence on the characteristic length of the ultraviolet behavior do not appear in the ratio. We may try to estimate this ratio by the assumption that ξ varies slowly to 1 when the k -mode we are interested in crosses the horizons. Then we can neglect the ξ term in (77) and have only the dependence of λ at sound horizon crossing. Since $r \ll 1$ from the observations, $\lambda_* = 1 + 2r$ must be very close to 1^+ , which is in consistency with our former assumption.

VI. CONCLUSION AND DISCUSSION

In this paper, we clarified several issues in the Hořava-Lifshitz gravity. We first showed that the strong coupling issue may not be so serious as argued in the literature before. The basic point is that the diffeomorphism is only a good approximation even at IR. Taking into account of the projectability condition, the usual Stuckelberg trick could not be applied naively. From our discussion, it seems that the extra dynamical scalar degree of freedom could be decoupled naturally.

However, the theory may suffer from other pathologies. One concern is on the existence of the ghost excitation. We showed that as the perturbations around the flat spacetime, the scalar perturbation around the flat FRW universe could be a ghost in the parameter region $\frac{1}{3} < \lambda < 1$. The presence of the ghost mode is a serious challenge to the theory. We tried to avoid the dangerous parameter region by mildly modifying the Hořava-Lifshitz action. We kept only the most UV sensitive and IR sensitive terms. We discussed the classical evolution and the power spectra of scalar and tensor perturbations. We obtained scale invariant spectrum if the Hubble constant H does not change. We also calculated the tensor-scalar ratio, and found it could be small under reasonable condition.

The nature of the power spectra studied in this paper is purely gravitational. In particular, in the language of orthodox cosmology, the scalar perturbation is expected to set up the initial conditions and seed the anisotropy of large scalar structure in our universe. Some work has been done to reveal the evolution of perturbations after inflation in Hořava-Lifshitz gravity[32]. After inflation ends, this gravitational perturbation must be converted into CMB anisotropy and matter inhomogeneity through some post-inflation evolutions. But still we do not know yet how to couple the gravitational scalar mode with, for instance, the radiation. This is an interesting issue, which we would like to study in future. Recently, an interesting paper on the Hořava-Lifshitz universe with single scalar field discussed the curvature perturbation ζ [34].

One essential issue in Hořava-Lifshitz gravity is on its RG flow. In [27], it has been shown that in Lifshitz-like scalar field theory, the RG flow may not lead the theory to the fixed point we want. Considering the numbers of the parameters in modified Hořava-Lifshitz gravity, this raise the concern if the theory can flow to IR fixed point $\lambda = 1$. Moreover, the details of RG flow can tell us if we can avoid the dangerous region, where the ghost excitation appears, even we start from a safe region. Furthermore, RG flow may closely related to the physics in the inflationary era. It is not clear whether RG flow of the theory runs to its IR fixed point before the inflationary era. If it did, then the gravitational scalar is not dynamical and has nothing to do with inflation. Even if the energy scale to reach IR limit is lower than the inflation era, there is an important question to answer: did λ vary significantly in the inflationary era? The variation of λ may tilt the power spectra and has interesting physical implications. In any case, the behavior of the Hořava-Lifshitz gravity theory under RG flow

deserves careful investigations.

Acknowledgments

The work was partially supported by NSFC Grant No.10535060, 10775002, 10975005 and RFDP. BC would like to thank R.G. Cai, Miao Li and Jian-xin Lu for stimulating discussions. BC also thank ICTS-USTC for hospitality, where the project was inspired. SP is grateful to Yi-Fu Cai and Tower Wang for useful directions.

-
- [1] P. Horava, “Membranes at Quantum Criticality,” JHEP **0903**, 020 (2009) [arXiv:0812.4287 [hep-th]].
 - [2] P. Horava, “Quantum Gravity at a Lifshitz Point,” Phys. Rev. D **79**, 084008 (2009) [arXiv:0901.3775 [hep-th]].
 - [3] E.M. Lifshitz, “On the Theory of Second-Order Phase Transitions I & II”, Zh. Eksp. Teor. Fiz **11** (1941)255 & 269.
 - [4] B. Chen and Q. G. Huang, “Field Theory at a Lifshitz Point,” arXiv:0904.4565 [hep-th].
 - [5] R. G. Cai, B. Hu and H. B. Zhang, “Dynamical Scalar Degree of Freedom in Horava-Lifshitz Gravity,” arXiv:0905.0255 [hep-th].
 - [6] B. Chen, S. Pi and J. Z. Tang, “Scale Invariant Power Spectrum in Hořava-Lifshitz Cosmology without Matter,” arXiv:0905.2300 [hep-th].
 - [7] C. Charmousis, G. Niz, A. Padilla and P. M. Saffin, “Strong coupling in Horava gravity,” arXiv:0905.2579 [hep-th].
D. Blas, O. Pujolas and S. Sibiryakov, “On the Extra Mode and Inconsistency of Horava Gravity,” arXiv:0906.3046 [hep-th].
 - [8] M. Fierz and W. Pauli, “On relativistic wave equations for particles of arbitrary spin in an electromagnetic field,” Proc. Roy. Soc. Lond. A **173**, 211 (1939).
 - [9] N. Arkani-Hamed, H. Georgi and M. D. Schwartz, “Effective field theory for massive gravitons and gravity in theory space,” Annals Phys. **305**, 96 (2003) [arXiv:hep-th/0210184].
 - [10] H. van Dam and M. J. G. Veltman, “Massive And Massless Yang-Mills And Gravitational Fields,” Nucl. Phys. B **22**, 397 (1970).
 - [11] H. Lu, J. Mei and C. N. Pope, “Solutions to Horava Gravity,” arXiv:0904.1595 [hep-th].
 - [12] A. Kehagias and K. Sfetsos, “The black hole and FRW geometries of non-relativistic gravity,” Phys. Lett. B **678**, 123 (2009) [arXiv:0905.0477 [hep-th]].
 - [13] J. Z. Tang and B. Chen, “Static Spherically Symmetric Solutions to modified Horava-Lifshitz Gravity with Projectability Condition,” arXiv:0909.4127 [hep-th].
 - [14] T. Harko, Z. Kovacs and F. S. N. Lobo, “Testing Hořava-Lifshitz gravity using thin accretion disk properties,” Phys. Rev. D **80**, 044021 (2009) [arXiv:0907.1449 [gr-qc]].
T. Harko, Z. Kovacs and F. S. N. Lobo, “Solar system tests of Hořava-Lifshitz gravity,” arXiv:0908.2874 [gr-qc].
L. Iorio and M. L. Ruggiero, “Horava-Lifshitz gravity and Solar System orbital motions,” arXiv:0909.2562 [gr-qc].
 - [15] M. Li and Y. Pang, “A Trouble with Hořava-Lifshitz Gravity,” arXiv:0905.2751 [hep-th].

- [16] Y. W. Kim, H. W. Lee and Y. S. Myung, “Nonpropagation of scalar in the deformed Hořava-Lifshitz gravity,” arXiv:0905.3423 [hep-th].
- [17] G. Calcagni, “Cosmology of the Lifshitz universe,” arXiv:0904.0829 [hep-th].
- [18] E. Kiritsis and G. Kofinas, “Horava-Lifshitz Cosmology,” arXiv:0904.1334 [hep-th].
- [19] S. Mukohyama, “Scale-invariant cosmological perturbations from Horava-Lifshitz gravity without inflation,” arXiv:0904.2190 [hep-th].
- [20] R. Brandenberger, “Matter Bounce in Horava-Lifshitz Cosmology,” arXiv:0904.2835 [hep-th]. Y. S. Piao, “Primordial Perturbation in Horava-Lifshitz Cosmology,” arXiv:0904.4117 [hep-th]. X. Gao, “Cosmological Perturbations and Non-Gaussianities in Hořava-Lifshitz Gravity,” arXiv:0904.4187 [hep-th]. T. Sotiriou, M. Visser, S. Weinfurtner, “Phenomenologically viable Lorentz-violating quantum gravity”, arXiv:0904.4464 [hep-th]. S. Mukohyama, K. Nakayama, F. Takahashi and S. Yokoyama, “Phenomenological Aspects of Horava-Lifshitz Cosmology”, arXiv:0905.0055 [hep-th]. S.Kalyana Rama, “Anisotropic Cosmology and (Super)Stiff Matter in Hořava’s Gravity Theory”, arXiv:0905.0700 [hep-th]. Alex Kehagias, Konstadinos Sfetsos, “The black hole and FRW geometries of non-relativistic gravity”, arXiv:0905.0477 [hep-th]. E. N. Saridakis, “Horava-Lifshitz Dark Energy,” arXiv:0905.3532 [hep-th]. X. Gao, Y. Wang, R. Brandenberger and A. Riotto, “Cosmological Perturbations in Hořava-Lifshitz Gravity,” arXiv:0905.3821 [hep-th]. M. Minamitsuji, “Classification of cosmology with arbitrary matter in the Hořava-Lifshitz theory,” arXiv:0905.3892 [astro-ph.CO]. A. Wang and Y. Wu, “Thermodynamics and classification of cosmological models in the Horava-Lifshitz theory of gravity,” arXiv:0905.4117 [hep-th]. S. Nojiri and S. D. Odintsov, “Covariant Horava-like renormalizable gravity and its FRW cosmology,” arXiv:0905.4213 [hep-th]. M. i. Park, “The Black Hole and Cosmological Solutions in IR modified Horava Gravity,” arXiv:0905.4480 [hep-th]. Y. F. Cai and X. Zhang, “Primordial perturbation with a modified dispersion relation,” arXiv:0906.3341 [astro-ph.CO]. M. i. Park, “A Test of Horava Gravity: The Dark Energy,” arXiv:0906.4275 [hep-th]. Y. Lu and Y. S. Piao, “Scale Invariance from Modified Dispersion Relations,” arXiv:0907.3982 [hep-th]. C. Appignani, R. Casadio and S. Shankaranarayanan, “The Cosmological Constant and Horava-Lifshitz Gravity,” arXiv:0907.3121 [hep-th].
- [21] C. Bogdanos and E. N. Saridakis, “Perturbative instabilities in Horava gravity,” arXiv:0907.1636 [hep-th].
- [22] J. Martin and R. H. Brandenberger, “The Trans-Planckian Problem of Inflationary Cosmology”, Phys. Rev. D **63**, 123501(2001) [arXiv:hep-th/0005209].
- [23] J. Martin and R. H. Brandenberger, “The Corley-Jacobson dispersion relation and trans-Planckian inflation,” Phys. Rev. D **65**, 103514 (2002) [arXiv:hep-th/0201189].
- [24] R. H. Brandenberger and J. Martin, Mod. Phys. Lett. A **16**, 999 (2001) [arXiv:astro-ph/0005432].
- [25] T. Takahashi and J. Soda, “Chiral Primordial Gravitational Waves from a Lifshitz Point,” arXiv:0904.0554 [hep-th].
- [26] T. P. Sotiriou, M. Visser and S. Weinfurtner, “Quantum gravity without Lorentz invariance,” arXiv:0905.2798 [hep-th].
- [27] R. Iengo, J. G. Russo and M. Serone, “Renormalization group in Lifshitz-type theories,” arXiv:0906.3477 [hep-th].
- [28] G. Calcagni, “Detailed balance in Horava-Lifshitz gravity,” arXiv:0905.3740 [hep-th].
- [29] S. Koh, “Relic gravitational wave spectrum, the trans-Planckian physics and Hořava-Lifshitz gravity,” arXiv:0907.0850 [hep-th].

- [30] K. Yamamoto, T. Kobayashi and G. Nakamura, “Breaking the scale invariance of the primordial power spectrum in Horava-Lifshitz Cosmology,” *Phys. Rev. D* **80**, 063514 (2009) [arXiv:0907.1549 [astro-ph.CO]].
- [31] A. Wang and R. Maartens, “Linear perturbations of cosmological models in the Horava-Lifshitz theory of gravity without detailed balance,” arXiv:0907.1748 [hep-th].
- [32] T. Kobayashi, Y. Urakawa and M. Yamaguchi, “Large scale evolution of the curvature perturbation in Horava-Lifshitz cosmology,” arXiv:0908.1005 [astro-ph.CO].
- [33] S. Carloni, E. Elizalde and P. J. Silva, “An analysis of the phase space of Horava-Lifshitz cosmologies,” arXiv:0909.2219 [hep-th].
- [34] A. Wang, D. Wands and R. Maartens, “Scalar field perturbations in Horava-Lifshitz cosmology,” arXiv:0909.5167 [hep-th].